# Math421 Group Theory: Assignment 2 February 2010

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

1. Assume that $H \cap K$ is a subgroup and prove it is normal if and only if both $H$ and $K$ are normal subgroups. Give an example of a non-trivial non-normal $H \cap K$ from the groups we have encountered so far. [5]

2. Your permuation $p$ will be the following throughout this question: (     ). Recall that the group $S_n$ of all permutations of the numbers $\{1, 2, ..., n\}$ has cardinality $n!$.

   (a) Find each of the elements of $S_4$ as a combination of $p$ and $q := (1234)$. [6]

   (b) Find a subgroup of each possible cardinality in $S_4$, but explain why there is no subgroup of cardinality 6 in $A_4$. [4]

   (c) Which subgroup is the centraliser of $p$ in $S_5$? [1]

3. Let $T$ be the group of cardinality 12 which has this presentation:

$$< x, y \mid x^4 = e,\ y^3 = e,\ yx = xy^2 >$$

   (a) Using subgroups and orders explain why $T$ is different from all the other four groups we know about with 12 vertices. [2]

   (b) Using $H :=< y >$, verify it is a normal subgroup and find which group $T/H$ is isomorphic to. [3]

   (c) Give a homomorphism $f$ from $T$ to $C_2 \times C_4$ which has $H$ as the kernel and use $K := Z(T)$ to verify the second isomorphism theorem in this case. [4]