Math421 Group Theory: Assignment 3 March 2010

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

1. Let your group G have the following presentation:

$$< s, t \mid s^8 = e, t^2 = e, st = ts^3 >$$

   (a) Find a normal subgroup K of cardinality 4 in G which involves both s and t and list the left cosets of K, and check the right cosets are equal. [3]

   (b) Draw a Cayley diagram for your group, checking the orders of 4 non-conjugate elements by tracing them around the diagram. [4]

   (c) Find a cardinality 8 subgroup H which contains K and check the third isomorphism theorem with your G, H and K. [4]

2. (a) Use the class equation to prove that all groups which have cardinality the square of a prime must be Abelian, and give several such examples. [6]

   (b) Find a non-Abelian group with a prime power cardinality, ensuring that your group is different from everyone else in the class and determine the sizes of its conjugacy classes and the centralisers corresponding to them. [3]

3. Let M be the group of 2×2 non-singular matrices with entries from $\mathbb{R}$ under matrix multiplication. Use matrix algebra to determine the conjugacy class of a matrix containing the last 4 digits of your registration number, giving some conjugates. [5]

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