Math421 Group Theory: Matt's Assignment 4 April 2010

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

1. Using your group from assignment 3, identify the conjugacy classes and verify the class equation. What do the Sylow theorems have to say about this group? [3]

2. Use the third Sylow theorem to prove that if $|G| = q^n m$ where q is prime and $m < q$ then the Sylow-q-subgroup is normal in G. Choose two odd primes p and q such that $p < q$ and p does not divide into $q - 1$ and use the above and Sylow's third theorem again to prove that G is a cyclic group. [7]

3. We are working in this question with a hexagonal necklace with 6 links and 6 beads. The group of the necklace's symmetries is D_6.

   (a) Find and list the number of rotationally different colourings of the links with n colours and (simultaneously) the beads with m colours. Verify your answers with $n = 1$ and $n = m = 2$. [6]

   (b) Use Burnside/Cauchy/Frobenius to count and list the colourings of the necklace with two colours such that there are just two green links and three green beads. [8]

Math421 Group Theory: Evan's Assignment 4 April 2010

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1. Using your group from assignment 3, identify the conjugacy classes and verify the class equation. What do the Sylow theorems have to say about this group? [3]

2. Use the third Sylow theorem to prove that if $|G| = q^n m$ where q is prime and $m < q$ then the Sylow-q-subgroup is normal in G. Choose two odd primes p and q such that $p < q$ and p does not divide into $q - 1$ and use the above and Sylow's third theorem again to prove that G is a cyclic group. [7]

3. We are working in this question with a hexagonal necklace with 6 links and 6 beads. The group of the necklace's symmetries is D_6.

   (a) Find and list the number of rotationally different colourings of the links with n colours and (simultaneously) the beads with m colours. Verify your answers with $n = 1$ and $n = m = 2$. [6]

   (b) Use Burnside/Cauchy/Frobenius to count and list the colourings of the necklace with two colours such that there is one green bead and an even number of green links. [8]