HMTH215
MTH215

UNIVERSITY OF ZIMBABWE

Bsc Honours Part II, General Part II/III Mathematics

Graph Theory

November 1997
Time : 2 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. Draw all the different trees which have seven vertices, indicating why they are all non-isomorphic. [8]

A2. Draw *all* simple graphs with valency sequence (5, 4, 4, 3, 2, 1, 1). [12]

A3. When is it possible that $G \circ H \approx G + H$? [8]

A4. Consider the graph with edge set

$$\{ab, ad, ah, ai, bc, be, bg, cd, ci, cj, de, di, dj, ef, eg, fg, fi, gh, hi, ij\}.$$

Show that it is non-planar by finding a subgraph of it homeomorphic to $K_{3,3}$. [12]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B8.

B5. (a) Define the graph theoretical concepts of chromatic index and cubic graphs. [4]

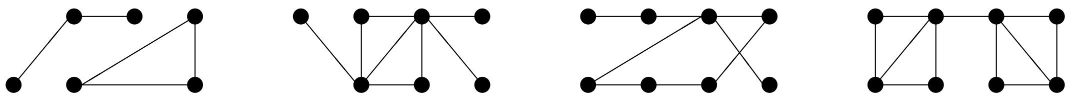
(b) Prove the value of the chromatic index of K_n. [10]

(c) Give examples of cubic graphs with 8, 9 and 10 vertices, if possible, proving why if not. [6]

(d) Is it true that a cubic Hamiltonian graph has chromatic index 3 ? Explain why. [10]

B6. (a) Prove that the chromatic polynomials of the graphs K_n and T_n (the complete graph and a tree on n vertices) are $\frac{t!}{((t-n)!}$ and $t(t-1)^{n-1}$ respectively. State the deletion-contraction formula and deduce from it the addition-identification formula. [14]

(b) Using the complete intersection formula and the formulas above find the chromatic polynomials of the four graphs shown below. [16]



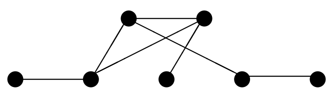
B7. (a) Draw the graph G which has this adjacency matrix after explaining how to get the valency sequence of G from the matrix. [5]

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

(b) Prove Euler's theorem categorising Eulerian graphs and the corollary about Euler paths. Hence deduce G is not Eulerian but has an Euler path. [20]

(c) Find and describe an Euler path in G, explaining how you did it. [5]

B8. (a) What are the eccentricities of the vertices in this graph ? Draw two other non-isomorphic graphs which have the same set of eccentricities. [8]



(b) Prove that the diameter of any graph is at most twice the radius and at least equal to the radius. [8]

(c) Give four graphs with diameter 6 and radii 3, 4, 5 and 6. Using evidence gained in this exercise or otherwise describe a way to draw a graph with diameter d and radius r for <u>any</u> valid r and d. [14]

END OF QUESTION PAPER