UNIVERSITY OF ZIMBABWE

HMTH215
MTH215

Bsc Honours/General Part II

Graph Theory

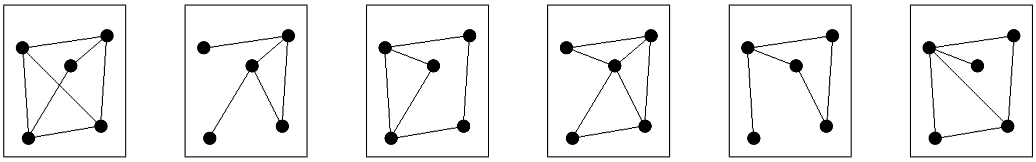
May 1999
Time : 2 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. Given this deck $D(G)$, show how you evaluate the valency sequence of G and hence, or otherwise, explain how to reconstruct G. [8]



A2. What is the valency sequence of the graph G defined below ?

$$\begin{aligned} V(G) &:= \{a,b,c,d,e,f,g,h,i,j\} \\ E(G) &:= \{bh,jh,ai,ed,jb,fi,jc,af,dg,bc\} \end{aligned}$$

Show how to create eight other graphs which have the same valency sequence as G; six should be connected and the other two should have exactly two components which are isomorphic to each other. Indicate why all graphs are non-isomorphic. [12]

A3. Embed K_6 on both the torus and the projective plane. In both cases count and label the faces in each embedding and hence verify Euler's characteristic for each surface. Also count the sum of the face sizes and check that they sum to twice the number of edges in K_6 in both embeddings. [10]

A4. Show that it is possible to create a self-complementary graph G with valency sequence (5,5,4,4,3,3,2,2) by first forming two copies of P_4 from the 5s and 2s and the 3s and the 4s. Give the mapping between G and $\overline{G}$. [10]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B8.

B5. (a) Carefully list all graphs with 5 vertices and at least 5 edges, giving the connectivity and chromatic number of each graph. [20]

(b) Prove that if G is a disconnected graph then $\overline{G}$ is connected. Show that there exists a connected graph with $n(\geq 4)$ vertices whose component is also connected. [10]

B6. (a) Evaluate the diameters and centres of these graphs: [12]

$$K_2 + C_5, \quad K_3 \times P_4, \quad C_5 \times C_3, \quad P_5 \circ K_2$$

(b) If $\text{diam}(G) = x$ and $\text{diam}(H) = y$, prove the general formulae for $\text{diam}(G + H)$, $\text{diam}(G \times H)$ and $\text{diam}(G \circ H)$ in terms of x and y. [18]

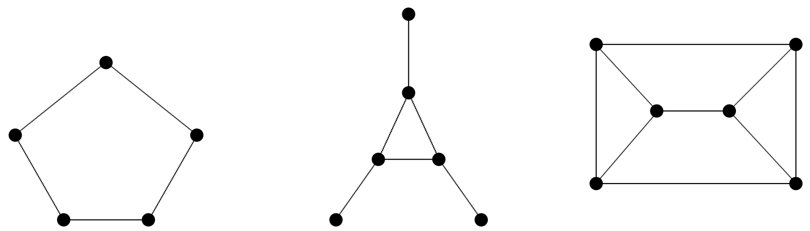
B7. (a) Using the "bridges and vertices of attachment" algorithm, and starting with cycle $vwrsyz$, find a planar embedding of the graph described by the following edge-set (clearly indicating each stage of the process by using separate diagrams and colours at all stages): [10]

$$\{rs, ut, qu, vw, uv, wr, sx, rx, st, uw, qv, zv, zt, zy, zw, zx, ty, sy, xy, xw\}$$

(b) Find an Eulerian path in the graph. Find a Hamiltonian cycle C and verify that Grinberg's theorem holds for the faces inside and outside C. [10]

B8. (a) Prove the deletion-contraction formula for chromatic polynomials and explain why the chromatic polynomial of a tree with n vertices is $t(t-1)^{n-1}$. Give the chromatic polynomials for K_3 and K_4. [12]

(b) Using the first part of the question and no other results from the course, find the chromatic polynomials of the following three graphs. [18]



END OF QUESTION PAPER